Prediction Material Model for Concrete – Rheology to Strength

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Outline

- McMaster University
- Motivation
- Introduction
- Packing Density
- Rheology Model
- Compressive Strength Model
- Conclusions

McMaster University

- One of the Top 100 universities in the world (Shanghai Jiao Tong University ranking of world universities and the Times Higher Education's Ranking of World Universities).
- In the 2012 Times Higher Education rankings of clinical, pre-clinical, and health universities, the university ranked 14th in the world and 1st in Canada.
- The "McMaster Model" – a student-centred, problem-based, interdisciplinary approach to learning – has been adopted by universities around the world.
- McMaster University ranks first in the country in research intensity--a measure of research income per full-time faculty member (Research Infosource Inc.)
- McMaster is the only North American host site of a United Nations University.
- McMaster participates in more than 40 international exchange agreements around the globe.
- McMaster is home to approximately 1,300 international students from 82 countries at the undergraduate level and approximately 700 students at the graduate level from 73 countries.

McMaster University

A Name Associated with Nobel Laurels

Some of Canada's most outstanding scientific minds were part of the McMaster community, including two Nobel prize winners: a McMaster Professor, Bertram Brockhouse and a McMaster graduate, Myron Scholes
Also, Dr. James Orbinski, a graduate from McMaster, received the Nobel Peace Prize on behalf of Doctors Without Borders

The Nobel Prize in Physics (1994)
Professor Bertram N.Brockhouse

The Nobel Prize in Economic (1997)
"McMaster gave me the freedom to think broadly and take calculated risks. McMaster set me on the correct trajectory and set the stage for my entire academic life".   - Myron Scholes

The Nobel Prize in Peace (1999)
Dr. James Orbinski

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Research Centres, Institutes and Facilities

- Antimicrobial Research Centre
- **Applied Dynamics Laboratory**
- Bertrand Russell Research Centre
- Brockhouse Institute for Materials Research
- CanChild Centre for Childhood Disability Research
- Canadian Cachrane Centre
- Centre for Advanced Polymer Processing & Design
- Centre for Electrophotonic Materials & Devices
- Centre for Gene Therapeutics
- Centre for Health Economics & Policy Analysis
- Centre for Minimal Access Surgery
- Microwave Acoustic Lab
- Research Institute for Quantitative Studies in Economics & Population
- Centre for Studies of Children at Risk
- Experimental Economics Laboratory
- Father Sean O' Sulivan Research Centre
- Firestone Institute for Respiratory Health
- Geographical Information Systems Lab
- Hamilton Civic Hospitals Research Centre
- Health & Social Services Utilization Research
- Health Information Research Unit
- Institute for Energy Studies
- Institute of Environment & Health
- Institute of Globalization

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Research Centres, Institutes and Facilities

- Centre for Peace Studies
- Centre for Pulp & Paper Research
- Centre for the Evaluation of Medicines
- Institute for the Study of Ancient & Forensic DNA
- Machine Vision & Image Analysis Lab
- Management of Innovation & New Technology
- McMaster Accelerator Lab
- McMaster Clinical Research Institute
- McMaster Nuclear Reactor
- McMaster Health Sciences International
- McMaster Manufacturing Research Institute
- R.Samuel McLaughlin Centre for Gerontological Health Research
- Neurocomputing for Signal Processing Lab
- Nursing Effectiveness Research Unit
- Research Centre for the Promotion of Women's Health
- Institute for Molecular Biology & Biotech
- Institute for Polymer Production Technology
- Institute for Population Health
- Steel Research Centre
- Supportive Cancer Care Research
- Water Resources Research /
- Environmental Information Systems
- William J. McCallion Planetarium
- Power Research Lab

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The International Connection

McMaster has always maintained strong international connections with international students from over 70 countries.

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Faculty of Engineering

McMaster's Faculty of Engineering, the most research-intensive engineering faculty in Canada, offers 4 year programs leading to the Bachelor of Engineering degree in the following field of specialization:

- Chemical Eng.
- Civil Eng.
- Engineering and Health Science
- Chemical & Bioengineering Eng.
- Electrical Eng.
- Software and Game Design
- Electrical & Biomedical Eng
- Engineering Physics
- Materials Eng.
- Mechanical Eng.
- Mechatronics Eng.
- Software Eng.
- Computer Eng.

5 year programs for Engineering & Management and Eng. & Society

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Admission Requirements

Graduate:

- Master's Degree

  The holding of an Honours bachelor's degree with at least B+ average in the final year in all courses in the discipline from recognized universities.

- Ph.D. Degree

  Completed a Master's program

- Strong letters of recommendation are required.
- English Language Requirements

  TOEFL score of at least 580 or 92

  (237 computerized)

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Motivation

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Sustainability

- Limited resources
- Waste
- Healthier environment
- $$ to invest

Provide the construction industry with a predictive tool for designing the hardened properties of concrete (Strength & Durability) on the basis of its constituents while using the fresh rheological properties as a quality control measure

Quality control improves the sustainability of the cement industry

Introduction

Historically, concrete mixture is proportioned to meet 3 design requirements:

- Workability → mixing, placing, consolidation, finishing
- Strength → compressive strength
- Durability → Air content, Cover

Workability: Design targets are slump and slump flow → necessary but not sufficient to quantify workability ➔ Rheology which is the science dealing with the deformation & flow of matter (liquid)

Compressive Strength: current design targets w/c → necessary but not complete as need to directly account for packing density and other factors should be included for better representation

Packing density

- Packing density, an indicator of how efficiently particles fill a certain volume, is defined as the ratio of the volume of the solid particles to the bulk volume occupied by these particles

$$Packing_{agg} = \frac{V_{agg}}{V_t} = \frac{V_{agg}}{V_{agg} + V_v} = 1 - e$$

ASTM C29

$$\phi^*_{agg} = \frac{V_{agg}}{V_t} = \frac{W_{agg}}{V_t \gamma_{agg}}$$

- Advantages → Denser packing through improved aggregates gradation and proportions, results in:

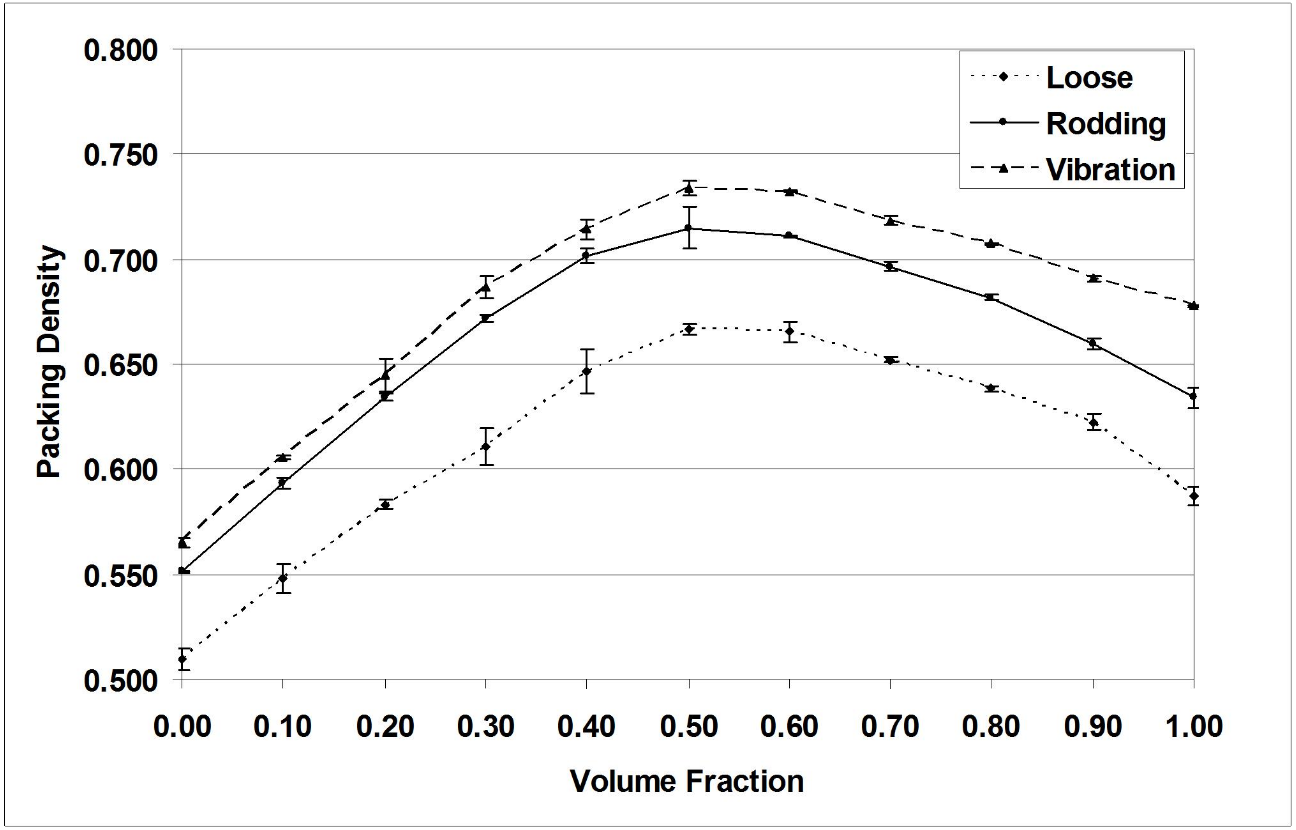
  - improved workability

  - higher strength and durability

Numerous packing models: What model is most representative for concrete applications? → Evaluation of models is needed

Evaluation of Packing Density Models

- Nine packing density models were evaluated for concrete applications.
- Experiments: aggregate volume fractions, methods of compaction



Packing Models	Loose	Rodding	Vibration
CPM_{error}	3.4%	1.6%	2.1%
MTM_{error}	4.9%	2.5%	3.5%
TPM_{error}	7.7%	3.6%	4.0%
$LMPM_{error}$	11.1%	7.8%	8.7%
$MLPM_{error}$	16.0%	14.0%	14.6%
LPM_{error}	17.0%	15.0%	15.5%
AGM_{error}	19.2%	17.2%	17.6%
$Furnas_{error}$	20.8%	17.2%	17.6%

- The Compressible Packing Model (CPM) [de Larrard, 1999] was found to be the most representative model for predicting the packing density of aggregates used in concrete

Workability and Rheology

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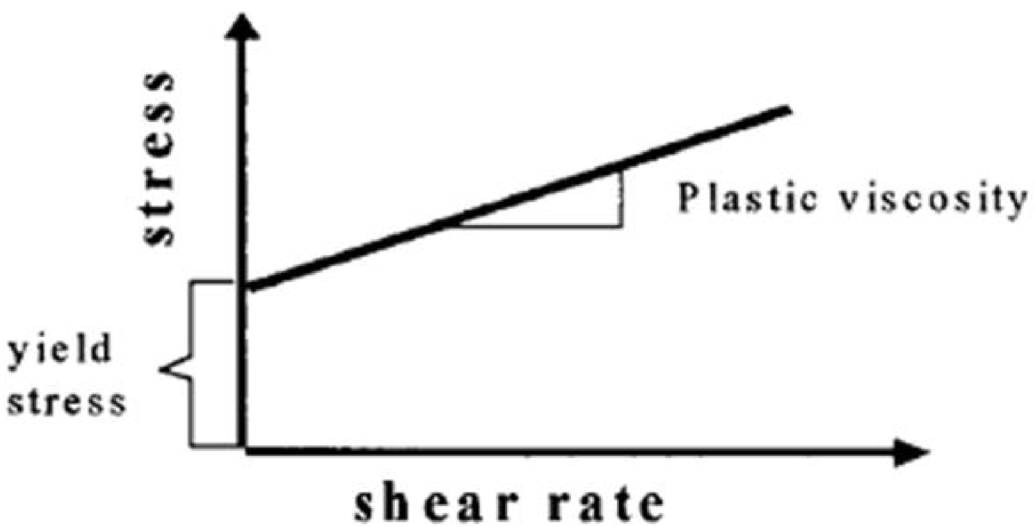
The American Concrete Institute (ACI) defines workability as: "the property of freshly mixed concrete or mortar which determines the ease with which it can be mixed, placed, compacted, and finished".

- Workability is qualitative / descriptive. The most commonly used tool to predict workability is the Slump test.
- Rheology is quantitative / fundamental and is a tool to quantify the workability of concrete

Why Rheology?

Quantification of the concrete flow behavior has become critical with new technologies that afford rapid placement of high quality concrete

- Ease of pumping are controlled by limiting the plastic viscosity
- SCC which flows under its own weight requires control of plastic viscosity and yield stress.



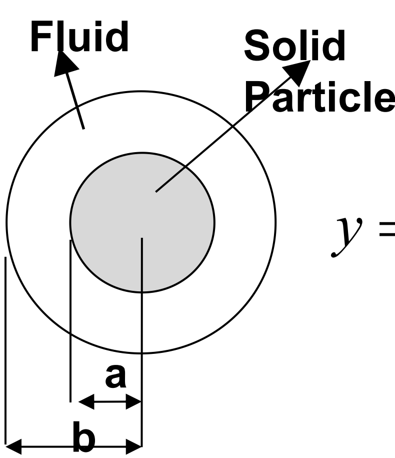
Rheology Model

Postulation

a) the shear stress is the sum of three components; static interaction between particles, dynamic interaction between particles, and collision of particles, and

$$\tau = \tau_0 + \tau_{DI} + \tau_{\text{collisions}}$$

b) the cell is a representative volume of the mixture.



$$y = \frac{a}{b}$$

Rheology Model

- Static yield stress

$$\tau_0 = \tau_i y(\varphi)^3 \frac{4(1 - y(\varphi)^7)}{4(1 + y(\varphi)^{10}) - 25y(\varphi)^3(1 + y(\varphi)^4) + 42y(\varphi)^5}$$

$$y(\varphi) = (\varphi/\varphi_{\max})^{1/3}\left(1 - \mathrm{C_Y}\left(\frac{m_G}{m_W} + \rho_w \cdot V_{air}\right)\right)$$

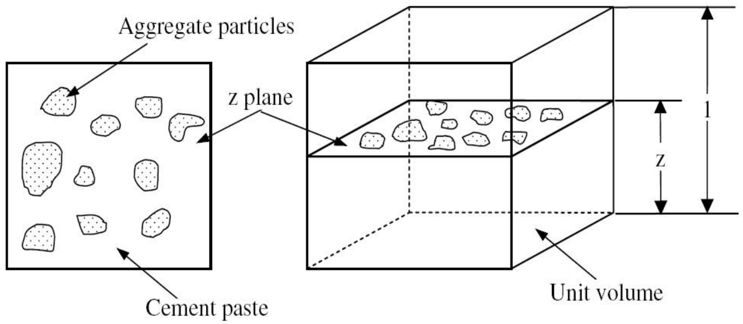
- Dynamic interaction

$$\tau_{DI} = \eta_w \cdot \eta_i \cdot y(\varphi)^3 \cdot \frac{4 \cdot \left(1 - y(\varphi)^7\right)}{4 \cdot \left(1 + y(\varphi)^{10}\right) - 25 \cdot y(\varphi)^3 \cdot \left(1 + y(\varphi)^4\right) + 42 \cdot y(\varphi)^5} \cdot \dot{\gamma}$$

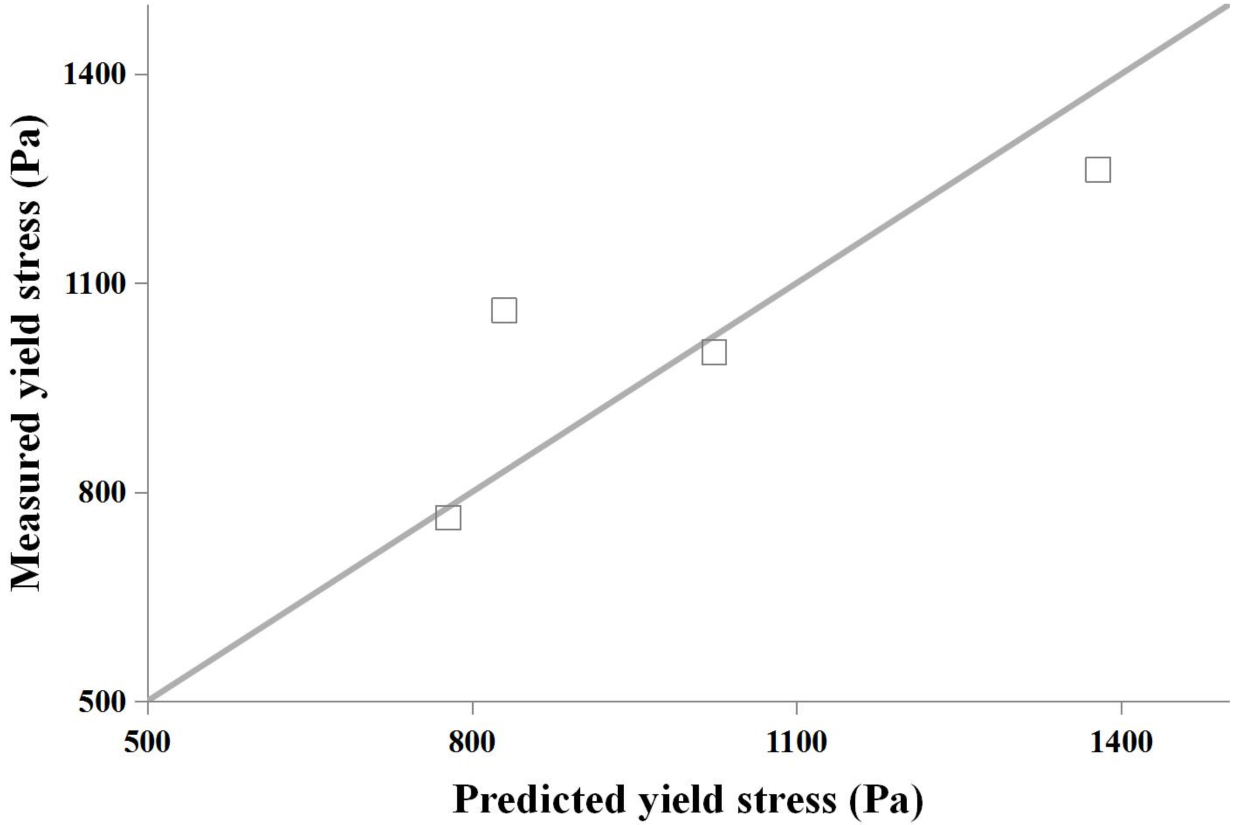
$$y(\varphi) = (\varphi/\varphi_{\max})^{1/3} \cdot \left(1 - C_p \frac{m_C}{m_W}\right)$$

- Particle collisions

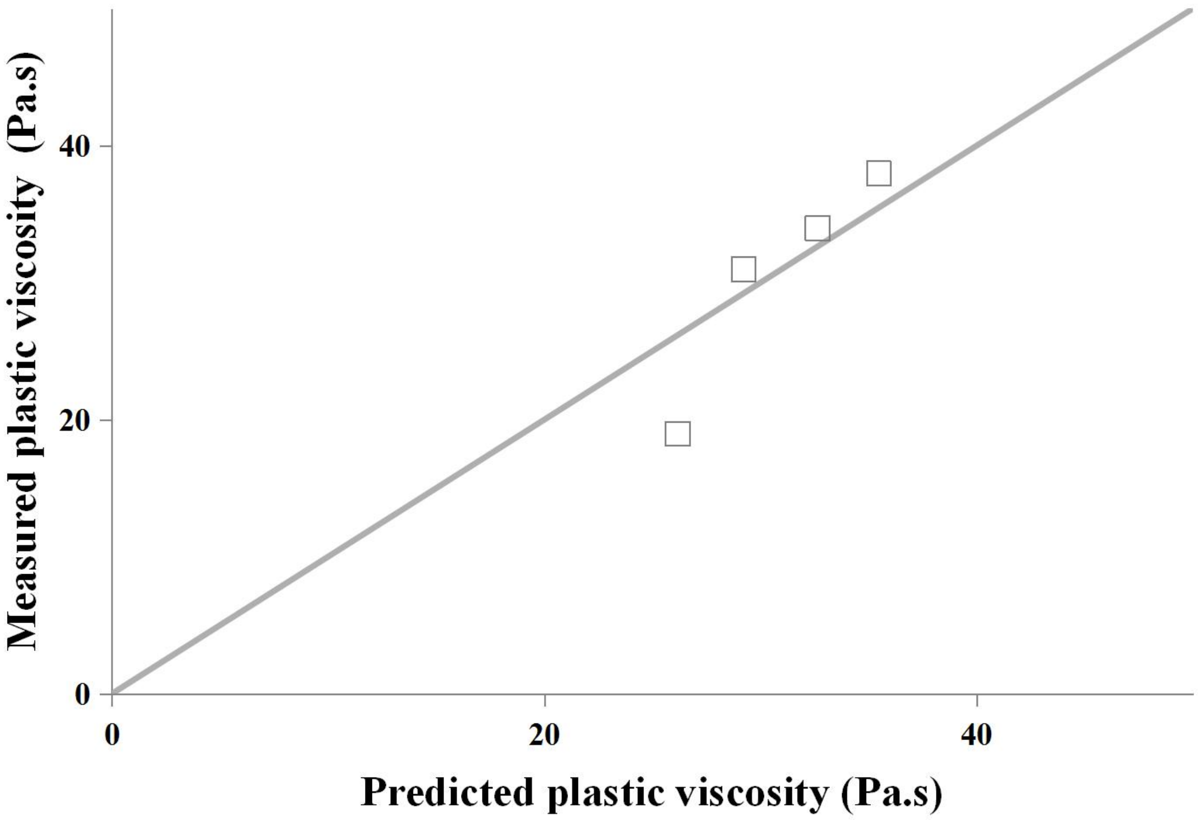
$$\tau_{\text{collisions}} = N_{collision}\left(k_p . F_{AP} + \Delta P\right)$$



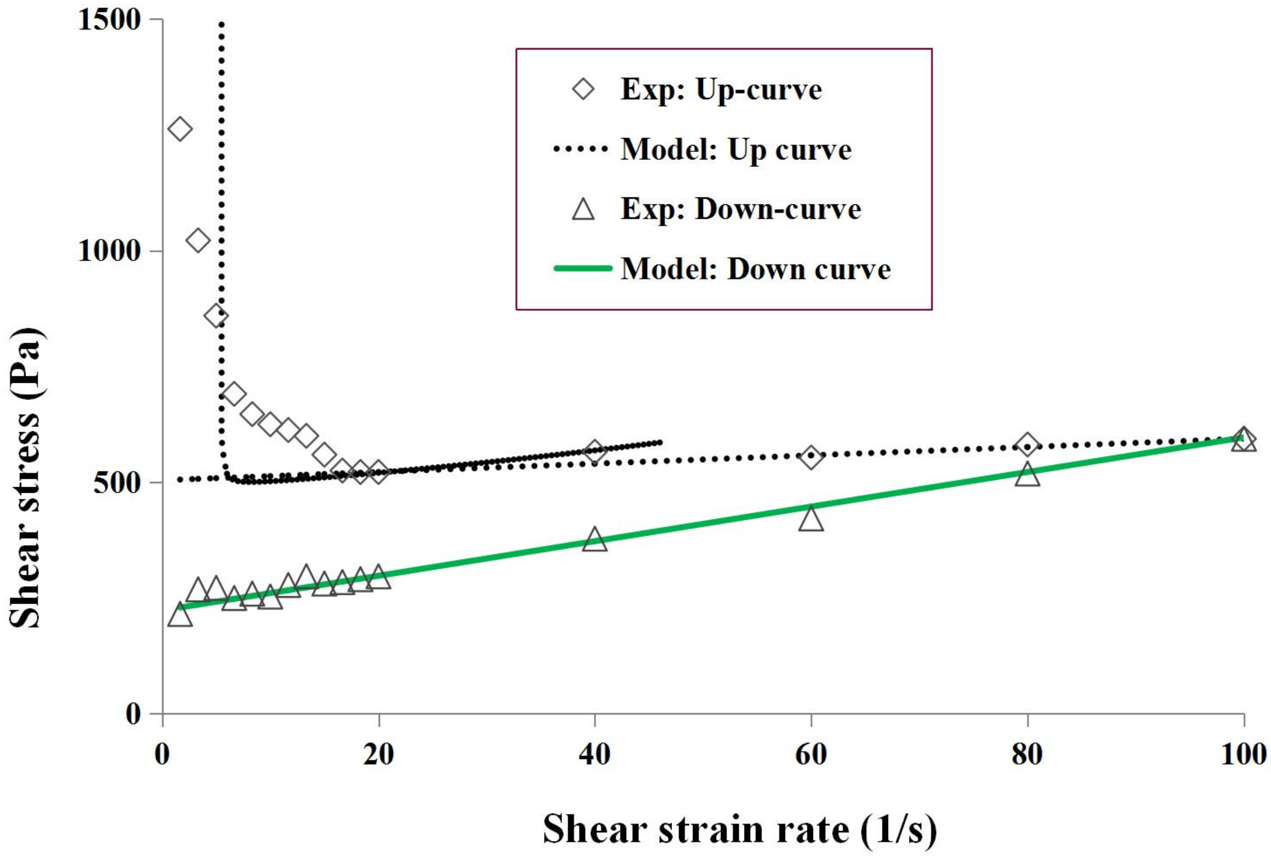
Concrete Yield Stress



Concrete Plastic Viscosity



Mortar Flow Response



Compressive Strength Model

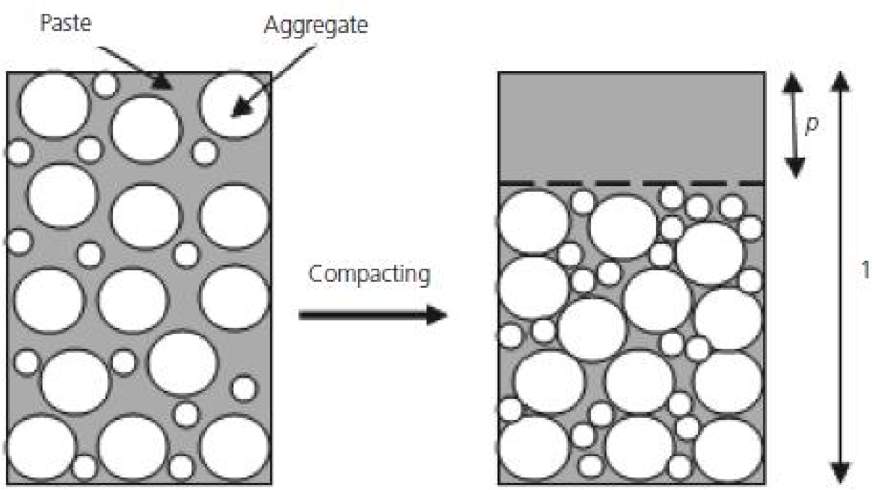
Compressive strength model comprises four submodels:

- Average paste thickness (APT) model → accounts for concrete mixture proportions and packing density;
- Cement hydration model → chemical composition of cement and the degree of cement hydration (α);
- Bond strength model → paste to aggregate bond strength (K);
- Cement paste strength model at 28 days (R_{c28}) → cement type (de Larrard, 1999).

$$f'_c(t) = F(K, R_{c28}, \mathrm{APT}, \alpha(t), \mathrm{w/c}, V_a)$$

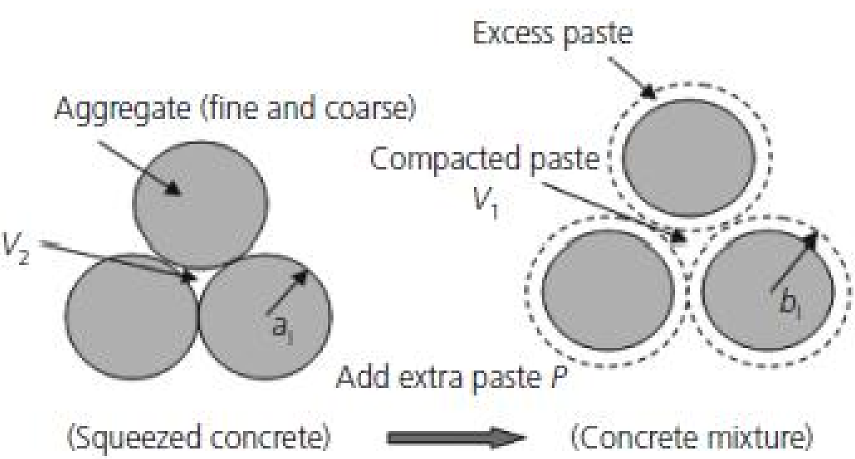
APT mathematical model

APT model Assumptions: excess paste theory, rigid noncohesive homogeneous mix, depends on particle shape



$$\frac{1-P}{\phi_s+\phi_{ca}}=\frac{1}{\phi_{max}}$$

$$\frac{1-P}{V_2}=\frac{1}{1-\phi_{max}}$$



$$P_e=P-(V_1-V_2)$$

$$\frac{V_1}{V_2}=\left(\frac{b}{a}\right)^3$$

$$\phi=\phi_s+\phi_{ca}=\frac{4}{3}\pi.a_s{}^3N_s+\frac{4}{3}\pi.a_{ca}{}^3N_{ca}$$

$$P_e=\frac{4}{3}\pi\left(b_s{}^3-a_s{}^3\right)N_s+\frac{4}{3}\pi\left(b_{ca}{}^3-a_{ca}{}^3\right)N_{ca}$$

APT Model (Cont)

Assuming:

$$APT_s = APT_{ca} = APT$$

$$APT = 2b - 2a = 2a\left(\frac{b}{a} - 1\right) = D\left(\frac{b}{a} - 1\right)$$

D= mean diameter of particles, a= particle radius, b= radius +excess paste

Final Result:

$$\frac{\phi_s}{D_s^{\ 3}}\left(APT + D_s\right)^3 + \frac{\phi_{ca}}{D_{ca}^{\ 3}}\left(APT + D_{ca}\right)^3 - \frac{\left(\phi - \phi / \phi^*\right)}{D^{\ 3}}\left(APT + D\right)^3 - 1 = 0$$

OR

$$APT \approx -\frac{1}{2}\left(D_s + \frac{\phi_{ca} D_s^{\ 2}}{\phi_s D_{ca}} + \frac{\phi D_s^{\ 2}\left(1 - \phi^*\right)}{\phi_s \phi^* D}\right) + \frac{1}{2}\sqrt{\left(D_s + \frac{\phi_{ca} D_s^{\ 2}}{\phi_s D_{ca}} + \frac{\phi D_s^{\ 2}\left(1 - \phi^*\right)}{\phi_s \phi^* D}\right)^2 + \frac{4}{3}\frac{\left(\phi^* - \phi\right)}{\phi^*}\frac{D_s^{\ 2}}{\phi_s}}$$

(cont.)

- For each concrete mixture, the maximum packing density, ϕ^*, can be calculated according to CPM.

- From statistical regressive analysis:

$$f_c' \propto \left(\frac{\text{APT}}{D}\right)^A$$

A depends on the shape of the particles

Cement hydration model

- Bi-linear relationships between f'_c and α
- Strength starts developing when α reaches the critical degree of hydration, α_{cr}

$$f'_c \propto (\alpha - \alpha_{cr})$$

- α_{cr} has been equated to the product of w/c and a constant 'm' equal to 0.43
- α can be determined using any cement hydration model → Schindler and Folliard (2005) model adopted

Capillary pores and air pores

- Capillary pores are controlled by w/c:
  increase w/c → increase capillary pores → decrease *f'c*
- Air pores, entrapped or entrained, also has a negative effect on strength.
- To account for these factors, Popovics' model (1985), a modification of Abrams' model, was adopted.

$$f'_c \propto B^{(w+V_a)/c}$$

B depends on specimen shape and test conditions

Bond strength and cement strength

- de Larrard's (1999) approach is adopted to account for the aggregate type and cement type through:

  - Paste to aggregate bond strength constant, K

  - Standard cement strength at 28 days, R_{c28}

Compressive strength model

- Particles gradation & proportions were modeled using an APT mathematical model formulated for the first time in this study.
- Hydration process modeled through α and accounts for α_{cr}
- Capillary pores and air pores modeled according to Popovics
- K and R_{c28} modeled following de Larrard

$$f'_c(t) = \begin{cases} 0 & \alpha(t) \leq \alpha_{cr} \\ K R_{c28} \left(\frac{\mathrm{APT}}{D} \right)^{A} B^{(w+V_a/c)} (\alpha(t) - \alpha_{cr}) & \alpha(t) > \alpha_{cr} \end{cases}$$

Calibration and Validation procedure

- Calibration of constants A, B and K was achieved by minimizing the standard error (σ) between model predictions and measured experimental data.

$$\sigma = \sqrt{\frac{\sum_i \left\{ f'_{c\_model}(i) - f'_{c\_exp}(i) \right\}^2}{n-p}}$$

Where ***n*** is the number of test points and ***p*** is the number of model constants

Experimental program

28 mixtures developed using Fractional Factorial Design

Variables: w/c: 0.4 to 0.7,

w: 175 to 228 kg/m^3,

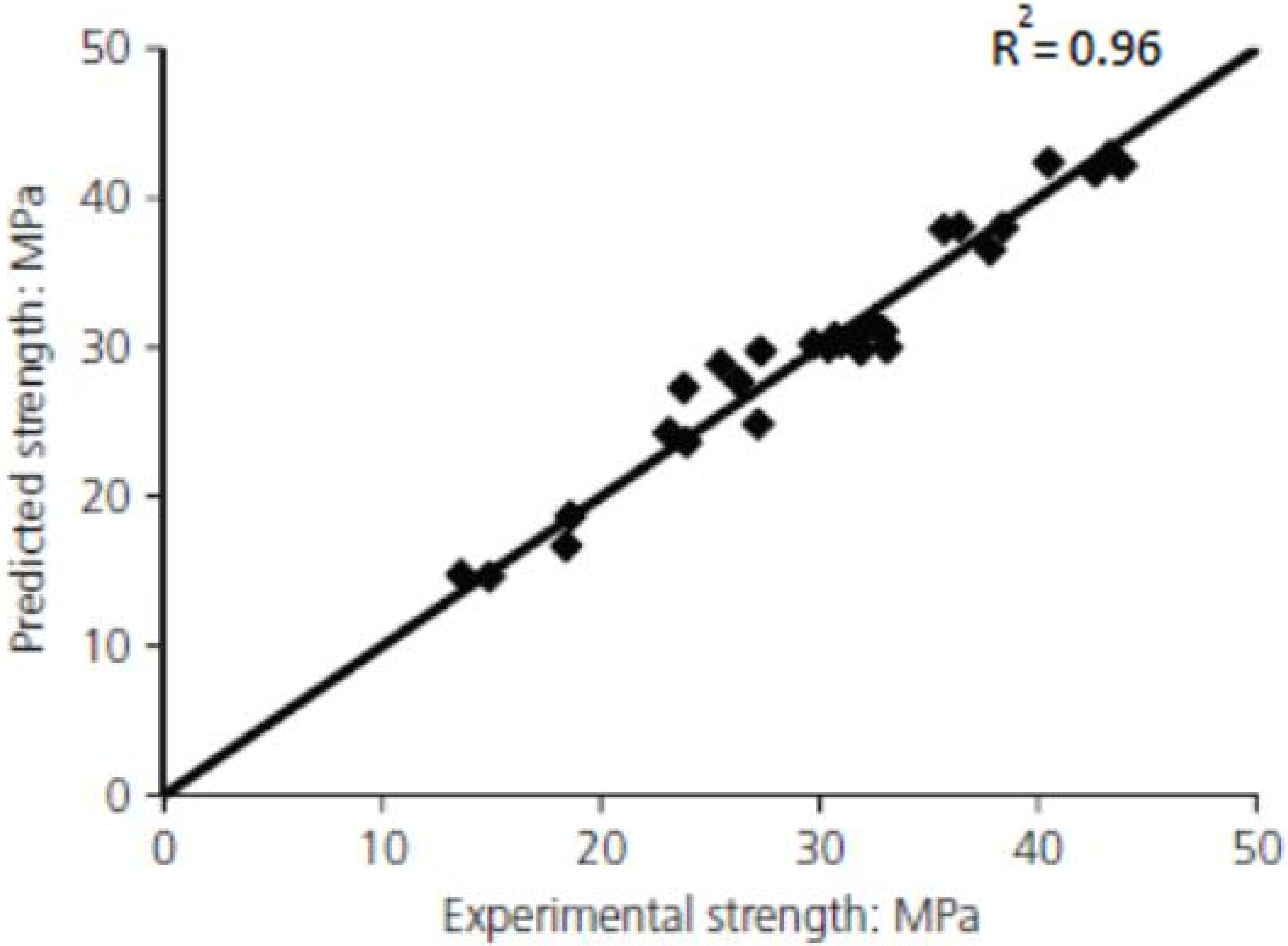
Max CA: 14 and 20 mm,

V_{CA}: 0.45 to 0.7,

Air Entrainment: 0% (8 mixes)

5% air (20 mixes)

Experimental results, calibration and validation



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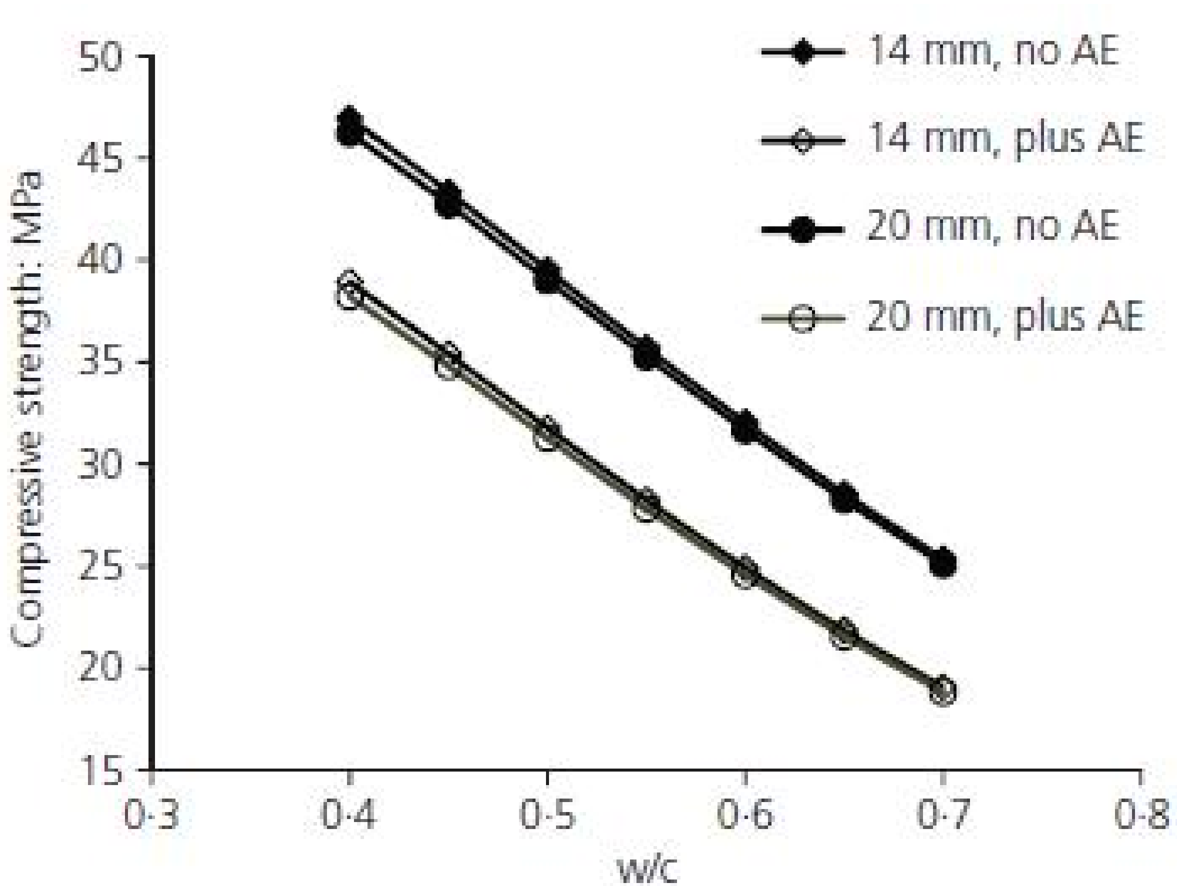

- 7 to 10 MPa difference between AE and non AE →consistent literature
- Increase in w/c → reduction in f'_c → consistent with literature
- Max CA size has minor effect on f'_c → consistent with literature

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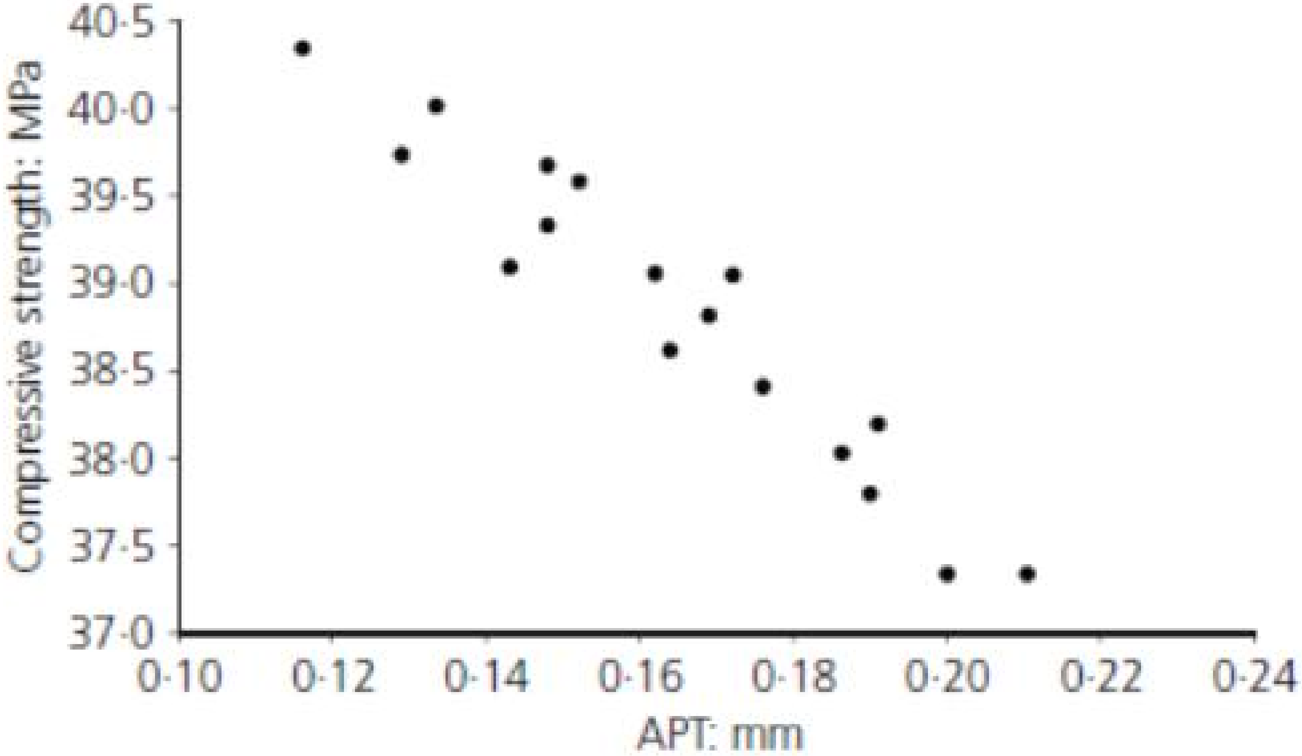

- Effect of APT on f'_c is around 7%
- Increase in APT → less compact mix→ shorter crack path due to fewer aggregates → lower f'_c

Age of concrete

- The degree of cement hydration is further developed:

$$f'_c = f_{28} \cdot \left(\frac{\alpha - \alpha_{cr}}{\alpha_u - \alpha_{cr}} \right)$$

- For OPC, the formulation is reworked as follows:

$$f'_c = f_{28} \cdot \left(\frac{430\,\mathrm{w/c} + 83 - 1031\,e^{-5 \cdot 3(1/t)^{0 \cdot 67}}}{430\,\mathrm{w/c} - 880} \right)$$

- The age extrapolation is found to accurately predict strength at 7 days and 191 days of experimental data reported in literature.

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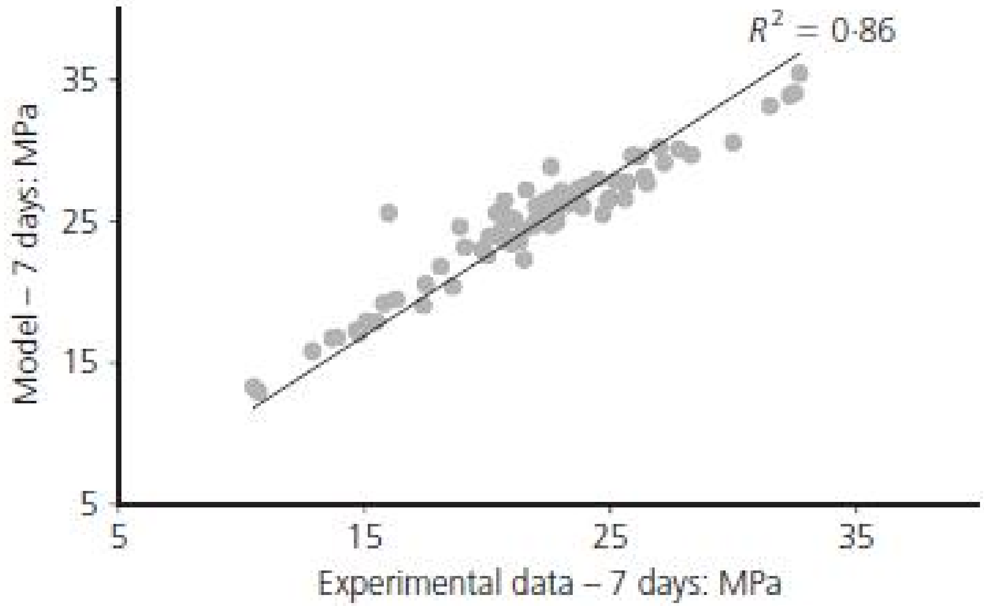

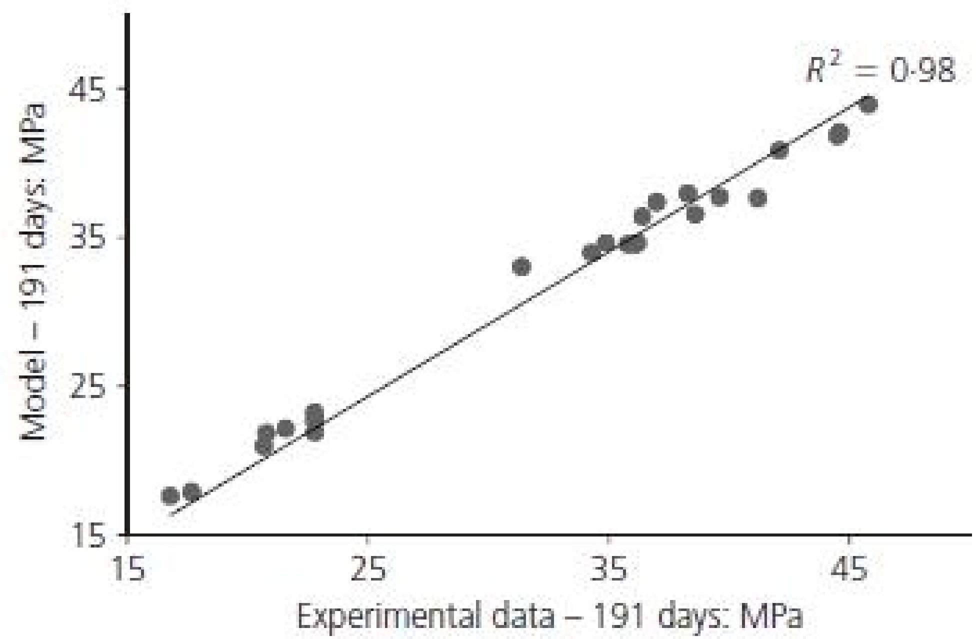

Comparative Analysis of Models

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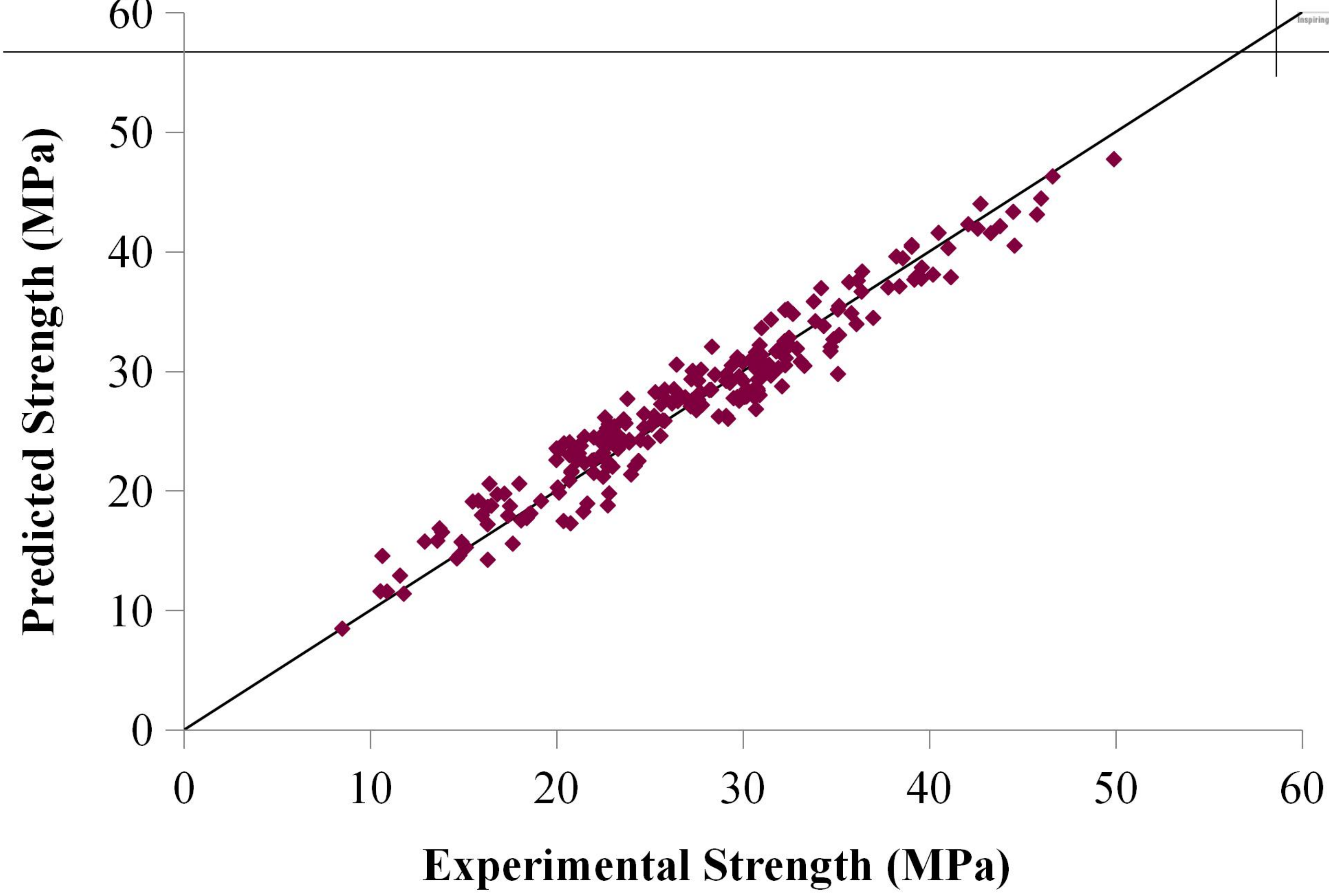
Assessment study using 250 measurements from 7 reported studies from literature (only few data pts were used for calibration)

Model	*p* (#)	σ (MPa)	R^2
Proposed Model (2013)	3	1.71	0.934
Mechling et al. (2009)	4	2.08	0.902
De Larrard (1999)	4	2.08	0.902
Pann et al. (2003)	4	2.80	0.843
Popovics (1998)	4	2.84	0.850
Popovics (2008)	4	2.85	0.850
Tango (2000)	4	2.86	0.837
Karni (1974)	2	2.86	0.833
Powers (1960)	2	2.96	0.824
Popovics (1985)	3	3.46	0.806
Feret (1892)	2	3.99	0.814



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Concluding Remarks

- Packing density links concrete mixture to rheology and strength.
- Rheological model is capable of predicting the rheological properties of mortar and fresh concrete.
- Experimental results support the postulation that the shear stress is the sum of static interaction, dynamic interaction and particles collision.
- For different ranges of shear strain rate, the governing constitutive equations of mortar are different and the proposed model can depict that behaviour.
- For normal slump concrete, it is proven fundamentally that the flow of fresh concrete obeys Bingham model.
- Shear stress due to collisions of particles is formulated to account for low and high Reynolds Number.
- Proposed model has the ability to provide a measure of thixotropy.

Concluding Remarks

- Compressive strength model comprises
  - APT model accounting for particles gradation and mixture proportions.
  - Cement hydration model accounting for the chemical composition and properties of the cementitious materials.
  - Paste to aggregate bond strength and cement standard strength.
  - Capillary pores and air pores.
- Strength model provides a good fit to the experimental data and does not contain outliers or discerning pattern.
- Standard error is less than 2 MPa for both 3-day and 28-day strength predictions.

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Questions

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Thank you